4.E: the ***PhilosophyIdeas*** overview of

Non-Classical Logics

Almost as soon as classical propositional and predicate logic had settled down, various difficulties arose which prompted an exploration of rival systems. The first doubts arose from mathematics, where huge infinities were being proposed, and a restriction to merely what can be proved was suggested. This led to **Intuitionistic Logic**, in which the concept of truth is replaced by 'provability'. Two immediate casualties from classical logic were the law of excluded middle (p v ¬p), and the rule of double negation elimination (¬¬p → p). In the case of a hypothesis it may not be possible to prove whether it is or is not the case. In the case of double negation, it may not possible to prove p by saying that it is 'not not-true' (as in 'I'm not unhappy'). Many of the normal practices of classical logic, such as proving connectives in terms of one another, thus become impossible. The result is a cut-down version of classical logic, which restricts the more extravagant claims of mathematics, but offers a higher level of security because it insists on proofs. Sympathisers with intuitionistic logic say that it rightly questions Bivalence, the claim that all propositions are either true or false, because in cases of unproven hypotheses, or vague objects and collections, bivalence is too boldly realist (in claiming that even unknowns are determinately true or false). The intuitionists don't offer a third value, but simply marginalise some truth values as 'unknown'. Those with anti-realist sympathies, and those who want to tie linguistic meaning to experience, like this logic. Opponents mainly object to its restriction on the flourishing further reaches of mathematics, which are both successful and important. The logic suffered a setback when it was proved that arithmetic contains some truths which are unprovable.

A different strategy for dealing with the doubts about Bivalence was to introduce a third truth-value, lying between T and F, called something like 'indeterminate'. The three values can be assigned numbers, such as -1, 0, and +1. The simple truth tables of classical logic can then be adapted to more elaborate three-value versions. The commonest version is called **Kleene Logic**, which comes in **Strong** and **Weak** versions. In classical logic disjunction (p v q) is true if one of p or q is true. In Weak Kleene disjunction is 'indeterminate', even if one disjunct is true. Thus the weaker system has many 'truth-value gaps', where the output is neither true nor false. Strong Kleene disjunction is true if one disjunct is true, even if the other is indeterminate. The strong version has fewer gaps, and seems more intuitive, and that is the more widely used system. In the end a logic seeks results, but 'indeterminate' can be used in proofs (rather as arithmetic treats zero as a number). It differs from classical logic because excluded middle and non-contradiction are not necessarily true, if the inputs are indeterminate. Clearly intuitionists will dislike such use of the unproved 'indeterminate', but the system is used in discussions of truth-value gaps, and in logic about presuppositions (where the truth-value is not yet known).

The problem of vagueness troubled classical logic, and the usual response was to refuse to deal with anything vague. With the introduction of a third value between T and F, there emerged the option of many values, which could map the vagueness, and this gave rise to **Fuzzy Logic**. The basic fuzziness dealt with is of set membership, which is normally determinate, but can now come in degrees. Unfortunately fuzziness has then to be allowed into the other components of the formulas, which threatens to get out of control. There is an obvious attraction for fine grades of computation, if we can say something is 79% true, though what it means to be also 21% false is not quite clear. It may be that fuzzy logic is just probability theory, but at least it addresses the persistent problem of vagueness.

Another problem in classical logic is the meaning of the existential quantifier (∃), which seems to assert the existence of something, and yet can refer to non-existent things and fictions. One response to that is the creation of **Free Logic**, in which you can use empty names, and maybe have an empty domain of objects, neither of which are allowed in classical logic. Typically it is used to reason about non-existent objects. You may doubt the existence of the 'circular square', for example, but still want to say it is square.

A central issue for non-classical logic is the way we reason about conditional truths. In classical logic that is dealt with by the '→' connective, but p→q is standardly translated into the other connectives by saying that it is equivalent to ¬pvq. That is, if p is true then q must be true, and the only way q can fail to be true is if p is false, so either q is true or p is false, and that's it. The problem, though is that if you assert p→q and p is false, that makes q true. Hence 'if 2+2=5 then pigs can fly', which sounds dubious. The principle says a falsehood implies everything, but how can p imply something about q if they have nothing in common? Hence we introduce **Relevant Logic**, which tries to isolate the real implications in the world, rather than the purely formal ones. We will now say 'q because of p', rather just 'q is implied by p'. Thus the logic must refer to 'situations' to support the implications, rather as intuitionists appealed to 'proofs'. The logic is basically classical, but with the addition that the conclusion must be somehow 'in' the premises. One nice result is that p no longer implies itself (since we don't say 'p because of p'!).

Many of these logics deal with truth-value 'gaps', but we may also be faced with truth-value 'gluts', where T and F overlap, and a sentence might count as both truth *and* false. If computers use classical logic in their programming they will crash when they meet a contradiction, but people don't crash in this way (usually!), because they work round the problem. We are familiar with answering some questions with 'well – yes and no', and many of our major theories contain paradoxes, so maybe we need a logic to cope with that. **Paraconsistent Logic** rejects the idea that if you accept a contradiction then every possible sentence is true (which does sound crazy!), and devises more sensible ways to cope. The aim is to distinguish different sorts of contradiction, and weigh their importance. Since contradictions can be tolerated certain standard reasonings are excluded (notably involving disjunctions, because you can no longer say that pvq and ¬p will necessitate q). An optional extension of the logic is to embrace **Dialethism**, which says reality itself contains contradictions.

Other non-classical logics include Intensional Logic, Awareness Logic, Justification Logic, Dynamic Logic, and non-monotonic systems. The biggest over-arching debate is whether there is 'one true logic', or whether we should be 'pluralists', and embrace different logics for different situations.